Chapter 2

Units and Dimensions

Units used to designate magnitude of a dimension have evolved based on common usage and instruments available for measurement. Two major systems for measurement have been used: the English system, which was used primarily in industry, and the metric system, which was used in the sciences. The confusion that results from the use of various terms to represent the same dimension has led to the development of a common system of units that is proposed for use in both science and industry. The *Système International d'Unites* (International System of Units) and the official international designation SI was adopted in 1960 by the General Conference on Weights and Measures. This body consists of delegates from member countries of the Meter Convention, and it meets at least once every 6 years. There are at least 44 countries represented in this convention, one of which is the United States.

The use of SI is now widespread in the scientific community. However, it is still often necessary to convert data from one system to another, as tables in handbooks may be in a different unit from what is needed in the calculations, or experimental data may be obtained using instruments calibrated in a different unit from what is desired in reporting the results.

In this chapter, the various units in SI are discussed, and techniques for conversion of units using the dimensional equation are presented. Also emphasized in this chapter is the concept of dimensional consistency of mathematical equations involving physical quantities and how units of variables in an equation are determined to ensure dimensional consistency.

2.1 DEFINITION OF TERMS

Dimension: used to designate a physical quantity under consideration (e.g., time, distance, weight).

Unit: used to designate the magnitude or size of the dimension under consideration (e.g., m for length, kg for weight).

Base unit: Base units are dimensionally independent. They are used to designate only one dimension (e.g., units of length, mass, and time).

Derived units: a combination of various dimensions. An example of a derived unit is the unit of force, which includes the dimensions of mass, length, and time.

Precision: synonymous with reproducibility, the degree of deviation of the measurements from the mean. This is often expressed as a $\pm$ term or as the smallest value of unit that can be consistently read in all determinations.

Table 2.1 Systems of Measurement.

	Dimension						
System	*Use*	*Length*	*Mass*	*Time*	*Temperature*	*Force*	*Energy*
English							
English absolute	Scientific	Foot	Pound mass	Second	°F	Poundal	BTU ft (poundal)
British Engineering	Industrial	Foot	Slug	Second	°F	Pound force	BTU ft (pound force)
American Engineering	U.S. industrial	Foot	Pound mass	Second	°F	Pound force	BTU ft (pound force)
Metric							
Cgs	Scientific	Centimeter	Gram	Second	°C	Dyne	Calorie, erg
Mks	Industrial	Meter	Kilogram	Second	°C	Kilogram force	Kilocalorie joule
SI	Universal	Meter	Kilogram	Second	°K	Newton	Joule

Accuracy: refers to how a measured quantity relates to a known standard. To test for accuracy of a measurement, the mean of a number of determinations is compared against a known standard. Accuracy depends on proper calibration of an instrument.

2.2 SYSTEMS OF MEASUREMENT

The various systems in use are shown in Table 2.1. These systems vary in the base units used. Under the English system, variations exist in expressing the unit of force. The chemical and food industries in the United States use the American Engineering System, although SI is the preferred system in scientific articles and textbooks.

The metric system uses prefixes on the base unit to indicate magnitude. Industry adopted the “mks” system, whereas the sciences adopted the “cgs” system. SI is designed to meet the needs of both science and industry.

2.3 THE SI SYSTEM

The following discussion of the SI system and the convention followed in rounding after conversion is based on the American National Standard, Metric Practice, adopted by the American National Standards Institute, the American Society for Testing and Materials, and the Institute of Electrical and Electronics Engineers.

2.3.1 Units in SI and Their Symbols

SI uses base units and prefixes to indicate magnitude. All dimensions can be expressed in either a base unit or combinations of base units. The latter are called derived units and some have specific names. The base units and the derived units with assigned names are shown in Table 2.2.

Table 2.2 Base Units of SI and Derived Units with Assigned Names and Symbols.

Quantity	*Unit*	*Symbol*[a]	*Formula*
Length	meter	m	–
Mass	kilogram	kg	–
Electric current	ampere	A	–
Temperature	kelvin	K	–
Amount of substance	mole	mol	–
Luminous intensity	candela	cd	–
Time	second	s	–
Frequency (of a periodic phenomenon)	hertz	Hz	1/s
Force	newton	N	kg m/s^2
Pressure, stress	pascal	Pa	N/m^2
Energy, work, quantity of heat	joule	J	N · m
Power, radient flux	watt	W	J/s
Quantity of electricity, electric charge	coulomb	C	A · s
Electric potential, potential difference, electromotive force	volt	V	W/A
Capacitance	farad	F	C/V
Electric resistance	ohm		V· A
Conductance	siemens	S	A/V
Magnetic flux	weber	Wb	V· s
Magnetic flux density	tesla	T	Wb/m^2
Inductance	henry	H	Wb/A
Luminous flux	lumen	1m	cd · sr[b]
Illuminance	lux	lx	1m/m^2
Activity (of radionuclides)	becquerel	Bq	1/s
Absorbed dose	gray	Gy	J/kg

Source: American National Standard, 1976. Metric Practice. IIEE Std. 268–1976. Institute of Electrical and Electronics Engineers, New York.
[a]Symbols are written in lowercase letters unless they are from the name of a person.
[b]sr stands for steradian, a supplementary unit used to represent solid angles.

2.3.2 Prefixes Recommended for Use in SI

Prefixes are placed before the base multiples of 10. Prefixes recommended for general use are shown in Table 2.3.

A dimension expressed as a numerical quantity and a unit must be such that the numerical quantity is between 0.1 and 1000. Prefixes should be used only on base units or named derived units. Double prefixes should not be used.

Examples:

1. 10,000 cm should be 100 m, not 10 kcm.
2. 0.0000001 m should be 1 μm.
3. 3000 m^3 should *not* be written as 3 km^3.
4. 10,000 N/m^2 can be written as 10 kPa but not 10 kN/m^2.

Table 2.3 Prefixes Recommended for Use in SI

Prefix	*Multiple*	*Symbol*[a]
tera	10^{12}	T
giga	10^{9}	G
mega	10^{6}	M
kilo	1000	k
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p
femto	10^{-15}	f

[a]Symbols for the prefixes are written in capital letters when the multiplying factor is 10^6 and larger. Prefixes designating multiplying factors less than 10^6 are written in lower case letters.

2.4 CONVERSION OF UNITS

2.4.1 Precision, Rounding-Off Rule, Significant Digits

Conversion from one system of units to another should be done without gain or loss of precision. Results of measurements must be reported such that a reader can determine the precision. The easiest way to convey precision is in the number of significant figures.

Significant figures include all nonzero digits and nonterminal zeroes in a number. Terminal zeroes are significant in decimals and they may be significant in whole numbers when specified. Zeroes preceding nonzero digits in decimal fractions are not significant.

Examples:

1. 123 has three significant figures.
2. 103 has three significant figures.
3. 103.03 has five significant figures.
4. 10.030 has five significant figures.
5. 0.00230 has three significant figures.
6. 1500 has two significant figures unless the two terminal zeroes are specified as significant.

When the precision of numbers used in mathematical operations is known, the answer should be rounded off following these rules recommended by the American National Standards Institute. All conversion factors are assumed to be exact.

1. In addition or subtraction, the answer shall not contain a significant digit to the right of the least precise of the numbers. Example: $1.030 + 1.3 + 1.4564 = 3.8$. The least precise of the numbers is 1.3. Any digit to the right of the tenth digit is not significant.
2. In multiplication or division, the number of significant digits in the answer should not exceed that of the least precise among the original numbers. Example: $123 \times 120 = 15,000$ if the terminal

zero in 120 is not significant. The least precise number has two significant figures and the answer should also have two significant figures.

3. When rounding-off, raise the terminal significant figure retained by 1 if the discarded digits start with 5 or larger, otherwise the terminal significant figure retained is unchanged. If the start of the digits discarded is 5, followed by zeroes, make the terminal significant digit retained even.

If the precision of numbers containing terminal zeroes is not known, assume that the number is exact. When performing a string of mathematical operations, round-off only after the last operation is performed.

Examples:

1253 rounded-off to two significant figures = 1300.
1230 rounded-off to two significant figures = 1200 (2 is even).
1350 rounded-off to two significant figures = 1400 (3 is odd).
1253 rounded-off to three significant figures = 1250.
1256 rounded-off to three significant figures = 1260.

2.5 THE DIMENSIONAL EQUATION

The magnitude of a numerical quantity is uncertain unless the unit is written along with the number. To eliminate this ambiguity, make a habit of writing both a number and its unit.

An equation that contains both numerals and their units is called a dimensional equation. The units in a dimensional equation are treated just like algebraic terms. All mathematical operations done on the numerals must also be done on their corresponding units. The numeral may be considered as a coefficient of an algebraic symbol represented by the unit. Thus, $(4\text{m})5 = (4)^2(\text{m})^2 = 16\text{m}5$ and

$$\left[5\frac{\text{J}}{\text{kg K}}\right](10\,\text{kg})(5\,\text{K}) = 5(10)(5)\left[\frac{\text{J kg K}}{\text{kg K}}\right] = 250\,\text{J}$$

Addition and subtraction of numerals and their units also follow the rules of algebra; that is, only like terms can be added or subtracted. Thus, $5\text{m} - 3\text{m} = (5 - 3)\text{m}$, but $5\text{m} - 3\text{cm}$ cannot be simplified unless their units are expressed in like units.

2.6 CONVERSION OF UNITS USING THE DIMENSIONAL EQUATION

Determining the appropriate conversion factors to use in conversion of units is facilitated by a dimensional equation. The following procedure may be used to set up the dimensional equation for conversion.

1. Place the units of the final answer on the left side of the equation.
2. The number being converted and its unit is the first entry on the right-hand side of the equation.
3. Set up the conversion factors as a ratio using Appendix Table A.1.
4. Sequentially multiply the conversion factors such that the original units are systematically eliminated by cancellation and replacement with the desired units.

Example 2.1. Convert BTU/(lb ≅ °F) to J/(g ≅ K)

$$\frac{\text{J}}{\text{g} \cdot \text{K}} = \frac{\text{BTU}}{\text{lb.°F}} \times \text{appropriate conversion factors}$$

The numerator J on the left corresponds to BTU on the right-hand side of the equation. The conversion factor is 1054.8 J/BTU. Because the desired unit has J in the numerator, the conversion factor must have J in the numerator. The factor 9.48×10^4 BTU/J may be obtained from the table, but it should be entered as J/9.48 $\times$ 10^4 BTU in the dimensional equation. The other factors needed are 2.2046×10^{-3} lb/g or lb/453.6 g and 1.8°E/K.

The dimensional equation is

$$\frac{\text{J}}{\text{g} \cdot \text{K}} = \frac{\text{BTU}}{\text{lb} \cdot {}^\circ \text{F}} \cdot \frac{1054.8 \text{ J}}{\text{BTU}} \cdot \frac{2.2046 \times 10^{-3}\text{lb}}{\text{g}} \cdot \frac{1.8^\circ\text{F}}{\text{K}}$$

Another form for the dimensional equation is

$$\frac{\text{J}}{\text{g} \cdot \text{K}} = \frac{\text{BTU}}{\text{lb} \cdot {}^\circ \text{F}} \cdot \frac{\text{J}}{9.48 \times 10^{-4}\text{BTU}} \cdot \frac{\text{lb}}{453.6\text{g}} \cdot \frac{1.8^\circ\text{F}}{\text{K}}$$

Canceling out units and carrying out the arithmetic operations:

$$\frac{\text{J}}{\text{g} \cdot \text{K}} = \frac{\text{BTU}}{\text{lb} \cdot {}^\circ \text{F}} \times 4.185$$

Example 2.2. The heat loss through the walls of an electric oven is 6500 BTU/h. If the oven is operated for 2 hours, how many kilowatt hours of electricity will be used just to maintain the oven temperature (heat input = heat loss)?

To solve this problem, rephrase the question. In order to supply 6500 BTU/h for 2 hours, how many kilowatt hours are needed? Note that power is energy/time; therefore the product of power and time is the amount of energy. Energy in BTU is to be converted to J.

The dimensional equation is

$$\text{J} = \frac{6500\,\text{BTU} \cdot 2\text{ h}}{\text{h}} \cdot \frac{1054.8\text{ J}}{\text{BTU}}$$

W = J/s, therefore W ≅ s = J,

$$\text{kW} \cdot \text{h} = \text{W} \cdot \text{s} \frac{1\text{kW}}{1000\text{W}} \frac{1\text{h}}{3600\text{s}}$$

$$\text{kW} \cdot \text{h} = \frac{6500\text{ BTU} \cdot 2\text{ h}}{\text{h}} \cdot \frac{1054.8\text{J}}{\text{BTU}} \cdot \frac{\text{kW}}{1000\text{W}} \cdot \frac{\text{h}}{3600\text{ s}}$$

$$\text{kW} \cong \text{h} = 3.809.$$

An alternative dimensional equation is

$$\text{kW} \cdot \text{h} = \frac{6500\,\text{BTU} \cdot 2\text{ h}}{\text{h}} \cdot \frac{\text{h}}{60\text{ min}} \cdot \frac{1.757 \times 10^{-2}\text{ kW}}{\text{BTU/ min}} = 3.809$$

The conversion factors in Appendix Table A.1 expresses the factors as a ratio and is most convenient to use in a dimensional equation. Although column I is labeled "denominator" and columns II and III are labeled "numerator," the reciprocal may be used. Just make sure that the numerals in column II go with the units in column III.

2.7 THE DIMENSIONAL CONSTANT (G_c)

In the American Engineering System of measurement, the units of force and the units of mass are both expressed in pounds. It is necessary to differentiate between the two units because they have actually different physical significance. To eliminate confusion between the two units, the pound force is usually written as lb_f and the pound mass is written as lb_m.

Because the force of gravity on the surface of the earth is the weight of a given mass, in the American Engineering System, the weight of 1 lb_m is exactly 1 lb_f. Thus, the system makes it easy to conceptualize the magnitude of a $1-lb_f$.

The problem in the use of lb_f and lb_m units is that an additional factor is introduced into an equation to make the units dimensionally consistent, if both units of force and mass are in the same equation. This factor is the dimensional constant g_c.

The dimensional constant g_c is derived from the basic definition of force: Force = mass × acceleration.

If lb_f is the unit of force and lb_m is the unit of mass, substituting ft/s^2 for acceleration results in a dimensional equation for force that is not dimensionally consistent.

$$lb_f = lb_m \cdot \frac{ft}{s^2}$$

The American Engineering System of measurement is based on the principle that the weight, or the force of gravity on a pound mass on the surface of the earth, is a pound force. Introducing a dimensional constant, the equation for force becomes:

$$lb_f = lb_m \cdot \frac{ft}{s^2} \cdot \frac{1}{g_c}$$

To make lb_f numerically equal to lb_m when acceleration due to gravity is 32.174 ft/s^2, the dimensional constant g_c should be a denominator in the force equation and have a numerical value of 32.174. The units of g_c is

$$g_c = \frac{ft \cdot lb_m}{lb_f \cdot s^2}$$

The dimensional constant, g_c, should not be confused with the acceleration due to gravity, g. The two quantities have different units. In SI, the unit of force can be expressed in the base units following the relationship between force, mass, and acceleration, therefore, g_c is not needed. However, if the unit kg_f is to be converted to SI, the equivalent g_c of $9.807\ kg \times m/kg_f \times s^2$ should be used. Use of conversion factors in Appendix Table A.1 eliminates the need for g_c in conversion from the American Engineering System of units to SI.

2.8 DETERMINATION OF APPROPRIATE SI UNITS

A key feature of SI is expression of any dimension in terms of the base units of meter, kilogram, and second. Some physical quantities having assigned names as shown in Table 2.2 can also be expressed in terms of the base units when used in a dimensional equation. When properly used, the coherent nature of SI ensures dimensional consistency when all quantities used for substitution into an equation are in SI units. The following examples illustrate selection of appropriate SI units for a given quantity.

Example 2.3. A table for viscosity of water at different temperatures lists viscosity in units of $lb_m/(ft \cong h)$. Determine the appropriate SI unit and calculate a conversion factor.

The original units have units of mass (lb_m), distance (ft), and time (h). The corresponding SI base units should be $kg/(m \cong s)$. The dimensional equation for the conversion is

$$\frac{kg}{m \cdot s} = \frac{lb_m}{ft \cdot h} \cdot \text{conversion factor} = \frac{lb_m}{ft \cdot h} \cdot \frac{1kg}{2.2046\ lb_m}$$

$$\frac{3.281ft}{1m} \cdot \frac{1h}{3600s} \cdot \frac{kg}{m \cdot s} = \frac{lb_m}{ft \cdot h}(3.84027 \times 10^{-5})$$

Viscosity in SI is also expressed in $Pa \cong s$. Show that this has the same base units as in the preceding example.

$$Pa \cdot s = \frac{N}{m^2} \cdot s = \frac{kg \cdot m}{s^2}\ \frac{s}{m^2} = \frac{kg}{m \cdot s}$$

Example 2.4. Calculate the power available in a fluid that flows down the raceway of a reservoir at a rate of 525 lb_m/min from a height of 12.3 ft. The potential energy (PE) is

$$PE = m\ g\ h$$

where m = mass, g = the acceleration due to gravity = 32.2 ft/s^2, and h is the height.

From Table 2.2, the unit of power in SI is the watt and the formula is J/s. Expressing in base units:

$$P = W = \frac{N \cdot m}{s} = \frac{kg \cdot m}{s^2} \cdot \frac{m}{s} = \frac{kg \cdot m^2}{s^3}$$

$$\frac{kg \cdot m^2}{s^3} = \frac{525lb_m}{min} \cdot \frac{32.2ft}{s^2} \cdot (12.3ft) \cdot \frac{kg}{2.2046\ lb_m} \cdot \frac{1\ min}{60s} \cdot \frac{m^2}{(3.281)^2 ft^2} = 146W$$

2.9 DIMENSIONAL CONSISTENCY OF EQUATIONS

All equations must have the same units on both sides of the equation. Equations should be tested for dimensional consistency before substitution of values of variables. A dimensional equation would easily verify dimensional consistency. Consistent use of SI for units of variables substituted into an equation ensures dimensional consistency.

Example 2.5. The heat transfer equation expresses the rate of heat transfer (q = energy/time) in terms of the heat transfer coefficient h, the area A, and the temperature difference ΔT.

Test the equation for dimensional consistency and determine the units of h.

$$q = h\ A\ \Delta\ T$$

The dimensional equation will be set up using SI units of the variables:

$$\frac{J}{s} = W = (-)(m^2)(K)$$

The equation will be dimensionally consistent if the units of h = W/m^2·K. Then m5 and K will cancel out on the right-hand side of the equation, leaving W on both sides, signifying dimensional consistency.

Example 2.6. Van der Waal's equation of state is as follows:

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

where P is pressure, n is moles, V is volume, R is gas constant, and T is temperature. Determine units of the constants a, b, and R and test for dimensional consistency.

Because the two terms in the left-hand side of the equation involve a subtraction, both terms in the first parenthesis will have units of pressure and in the second parenthesis units of volume. The product of pressure times volume will be the same units on the right side of the equation.

Using SI units for pressure of N/m^2 and volume, m^3, the dimensional equation is

$$\frac{N}{m^2}(m^3) = (\text{kg mole})(- - -)(^\circ K)$$

The equation will be dimensionally consistent if the units of R = N· m/kg mole because that will give the same units, Nm, on both sides of the equation. The units of a and b are determined as follows. The first term, which contains a, has units of pressure:

$$\frac{N}{m^2} = \frac{(\text{kg mole})^2(- - - -)}{(m^3)^2}$$

To make the right-hand side have the same units as the left, a will have units of $N(m)^4/(\text{kg mole})^2$. The second term, which contains b has units of volume.

$$m^3 = (\text{kg mole})(- - - -)$$

Thus, b will have units of m^3/kg mole.

2.10 CONVERSION OF DIMENSIONAL EQUATIONS

Equations may be dimensionless (i.e., dimensionless groups are used in the equation). Examples of dimensionless groups are Reynolds number (Re), Nusselt number (Nu), Prandtl number (Pr),

Fourier number (Fo), and Biot number (Bi). These numbers are defined below, and the dimensional equations for their units show that each group is dimensionless. The variables are as follows: V is velocity, D is diameter, ρ is density, μ is viscosity, k is thermal conductivity, h is heat transfer coefficient, α is thermal diffusivity, C_p is specific heat, L is thickness, and t is time.

$$Re = \frac{DV\rho}{\mu} = m\frac{m\,kg}{s\,m^3}\,\frac{1}{kg/(m\cdot s)}$$

$$Nu = \frac{hD}{k} = \frac{W}{m^2\cdot K}m\frac{1}{W/(m\cdot K)}$$

$$\mathrm{Pr} = \frac{C_\rho\,\mu}{k} = \frac{J}{kg \cdot K} \cdot \frac{kg}{m \cdot s} \cdot \frac{1}{J/(s \cdot m \cdot K)}$$
$$\mathrm{Fo} = \frac{\propto t}{L^2} = \frac{m^2}{s} \cdot s \cdot \frac{1}{m^2}$$
$$\mathrm{Bi} = \frac{hL}{k} = \frac{W}{m^2 \cdot K} \cdot m \cdot \frac{1}{W/(m \cdot K)}$$

An example of a dimensionless equation is the Dittus-Boelter equation for heat transfer coefficients in fluids flowing inside tubes:

$$\mathrm{Nu} = 0.023\,(\mathrm{Re})^{0.8}\,(\mathrm{Pr})^{0.3}$$

Because all terms are dimensionless, there is no need to change the equation regardless of what system of units is used in determining the values of the dimensionless groups.

Dimensional equations result from empirical correlations (e.g., statistical analysis of experimental data). With dimensional equations, units must correspond to those used on the original data, and substitution of a different system of units will require a transformation of the equation.

When converting dimensional equations, the following rules will be useful:

1. When variables appear in the exponent, the whole exponent must be dimensionless, otherwise it will not be possible to achieve dimensional consistency for the equation.
2. When variables are in arguments of logarithmic functions, the whole argument must be dimensionless.
3. Constants in an equation may not be dimensionless. The units of these constants provide for dimensional consistency.

Example 2.7. The number of surviving microorganisms in a sterilization experiment is linear in a semi-logarithmic graph, therefore, the equation assumes the form:

$$\log N = -a\,t + b$$

where N is number survivors, t is time, and a and b are constants. The equation will be dimensionally consistent if b is expressed as log N at time 0, designated log N_o, and a will have units of reciprocal time. Thus, the correct form of the equation that is dimensionally consistent is

$$\log \frac{N}{N_o} = -at$$

In this equation, both sides are dimensionless.

Example 2.8. An equation for heat transfer coefficient between air flowing through a bed of solids and the solids is

$$h = 0.0128\,G^{0.8}$$

where G is mass flux of air in lb/($ft^2 \cong$ h) and h is heat transfer coefficient in BTU/(h $\cong ft^2 \cong$ F). Derive an equivalent equation in SI.

The dimensional equation is

$$\frac{BTU}{h \cdot ft^2 \cdot F} = (- - -)\left(\frac{lb}{ft^2 \cdot h}\right)^{0.8}$$

An equation must be dimensionally consistent; therefore, the constant 0.0128 in the above equation must have units of

$$\frac{\mathrm{BTU}}{\mathrm{h\cdot ft^2\cdot F}}\left(\frac{\mathrm{ft^{1.6}h^{0.8}}}{\mathrm{lb^{0.8}}}\right)=\frac{\mathrm{BTU}}{\mathrm{h^{0.2}ft^{0.4}lb^{0.8}F}}$$

Converting the equation involves conversion of the coefficient. The equivalent SI unit for h = W/(m^2K) and G = kg/(m^2 ≅ s). Thus, the coefficient will have units of J/($s^{0.2}m^{0.4}kg^{0.8}$K). The dimensional equation for the conversion is

$$\frac{\mathrm{J}}{\mathrm{s^{0.2}m^{0.4}kg^{0.8}K}}=\frac{0.0128\ \mathrm{BTU}}{\mathrm{h^{0.2}ft^{0.4}lb^{0.8}F}}\cdot\frac{1054.8\ \mathrm{J}}{\mathrm{BTU}}\cdot\frac{\mathrm{ft^{0.4}}}{(0.3048)^{0.4}\mathrm{m^{0.4}}}$$

$$\frac{\mathrm{h^{0.2}}}{(3600)^{0.2}(\mathrm{s^{0.2}})}\cdot\frac{(2.2048)^{0.8}\mathrm{lb^{0.8}}}{\mathrm{kg^{0.8}}}\cdot\frac{1.8\ \mathrm{F}}{\mathrm{K}}=14.305$$

The converted equation is

$$h=14.305\,G^{0.8}$$

where both h and G are in SI units. To check: If G = 100 lb/(ft^2 ≅ h) and h = $0.0128(100)^{0.8}$, h = 0.5096 BTU/(h ≅ ft^2 ≅ °F).

The equivalent SI value is

$$\frac{0.5096\ \mathrm{BTU}}{\mathrm{h\cdot ft^2\cdot F}}\cdot\frac{5.678263\mathrm{W/(m^2\cdot K)}}{\mathrm{BTU/(h\cdot ft^2\cdot {}^\circ F)}}=2.893\frac{\mathrm{W}}{(\mathrm{m^2\cdot K})}$$

In SI, G = [100lb/(ft^2· h)][ft^2/0.3048$)^2m^2$](h/3600 s)(kg/2.2046 lb) = 0.1356. Using converted equation,

$$h=14.305\ (0.1356)0^{.8}=2.893\ \mathrm{W/(m^2\cong K)}.$$

PROBLEMS

2.1. Set up dimensional equation and determine the appropriate conversion factor to use in each of the following

$$\frac{\mathrm{lb}}{\mathrm{ft^3}}=\frac{\mathrm{lb}}{\mathrm{gal}}\times\ \text{conversion factor}$$

$$\frac{\mathrm{lb}}{\mathrm{in^2}}=\frac{\mathrm{lb}}{\mathrm{ft^2}}\times\ \text{conversion factor}$$

$$\mathrm{W}=\frac{\mathrm{cal}}{\mathrm{s}}\times\ \text{conversion factor}$$

2.2. The amount of heat required to change the temperature of a material from T_1 to T_2 is given by:

$$q=mC_p(T_2-T_1)$$

where q is BTU, m is mass of material in lb, C_p is specific heat of material in BTU/lb·°F, and T_1 and T_2 are initial and final temperatures in °F.

(a) How many BTUs of heat are required to cook a roast weighing 10 lb from 40°F to 130°F? Cp = 0.8 BTU/(lb ≅°F)

(b) Convert the number of BTUs of heat in (a) into watt-hours.

(c) If this roast is heated in a microwave oven having an output of 200 watts, how long will it take to cook the roast?

2.3. How many kilowatt hours of electricity will be required to heat 100 gallons of water (8.33 lb/gal) from 60°F to 100°F? C_p of water is 1 BTU/(lb $\cong$ °F).

2.4. 4. Calculate the power requirements for an electric heater necessary to heat 10 gallons of water from 70°F to 212°F in 10 minutes. Express this in Joules/min, and in watts. Use the following conversion factors in your calculations:

Specific heat of water = 1 BTU/(lb $\cong$ °F)
3.414 BTU/(W $\cong$ h)
60 min/h
3600 s/h
8.33 lb water/gal
1.054×10^3 J/BTU

2.5. One ton of refrigeration is defined as the rate of heat withdrawal from a system necessary to freeze 1 ton (2000 lb) of water at 32°F in 24 hours. Express this in watts, and in BTU/h. Heat of fusion of water = 80 cal/g.

2.6. (a) In the equation $\tau = \mu(\gamma)$, what would be the units of τ in the equation if μ is expressed in dyne $\cong$ s/cm^2 and γ is in s^{-1}?

(b) If μ is to be expressed in lb$_m$/(ft $\cong$ s), τ is expressed in lb$_f$/ft^2, and γ is in s^{-1}, what is needed in the equation to make it dimensionally consistent?

2.7. In the equation

$$\overline{V} = \frac{1000(\rho_1 - \rho_2)}{m\rho_1\rho_2} + \frac{M}{\rho_2}$$

what units should be used for the density, ρ, such that $\overline{V}$ would have the units ml/mole?

(a) m = moles/1000 g
(b) M = g/mole

2.8. Express the following in SI units. Follow the rounding-off rule on your answer.

(a) The pressure at the base of a column of fluid 8.325 in. high when the acceleration due to gravity is 32.2 ft/s^2 and the fluid density is 1.013 g/cm^3.

P = density $\times$ height $\times$ acceleration due to gravity

(b) The compressive stress (same units as pressure) on a specimen having a diameter of 0.525 in. when the applied force is 5.62 pound force.

Stress = Force/area

(c) The force needed to restrain a piston having a diameter of 2.532 in. when a pressure of 1500 (exact) lb$_f$/in^2 is in the cylinder behind the piston.

Force = Pressure $\times$ area

2.9. An empirical equation for heat transfer coefficient in a heat exchanger is

$$h = a(V)^{0.8}(1 + 0.011T)$$

where h = BTU/(h $\cong$ ft^2 $\cong$ Δ°F), V = ft/s, and T = °F. In one experimental system, a had a value of 150. What would be the form of the equation and the value of a if h, V, and T are in SI units?

2.10. A correlation equation for the density of a liquid as a function of temperature and pressure is as follows:

$$d = (1.096 + 0.0086\ T)(e)^{0.000953P}$$

where d is density in g/cm^3, T is temperature in Kelvin, P is pressure in atm, and e is the base of natural logarithms. A normal atmosphere is 101.3 kPa. Determine the form of the equation if all variables are to be expressed in SI.

2.11. The Arrhenius equation for the temperature dependence of diffusivity (D) is given by $D = D_o[e]^{-E/RT}$. R is a constant with a value of 1.987 cal/mole K. (T is temperature in degrees Kelvin). If D is in cm^2/s, determine the units of D_o and E.

2.12. The heat of respiration of fresh produce as a function of temperature is $q = a\ e^{bT}$. If q has units of BTU/(tonA24 h), and T is in °F, determine the units of a and b. The values of a and b for cabbage are 377 and 0.041, respectively. Calculate the corresponding values if q is expressed in mW/kg and T is in °C.

SUGGESTED READING

American National Standards Institute 1976. American National Standard, metric practice. Am. Natl. Stand. Inst. IIEE Std. 268–1976. IEEE, New York.

American Society for Agricultural Engineers 1978. Use of customary and SI (metric) units. American Society for Agricultural Engineers Yearbook—1978. ASAE, St. Joseph, MI.

Benson, S. W. 1971. Chemical Calculations. 3rd ed. John Wiley & Sons, New York.

Feldner, R. M. and Rousseau, R. W. 1999. Elementary Principles of Chemical Processes. 2nd ed. John Wiley & Sons, New York.

Himmelblau, D. M. 1967. Basic Principles and Calculations in Chemical Engineering. 2nd ed. Prentice-Hall, Englewood Cliffs, NJ.

Kelly, F. H. C. 1963. Practical Mathematics for Chemists. Butterworths, London.

McCabe, W. L., Smith, J. C., and Harriott, P. 1985. Unit Operations of Chemical Engineering. 4th ed. McGraw-Hill Book Co., New York.

Obert, E. and Young, R. L. 1962. Elements of Thermodynamics and Heat Transfer. McGraw-Hill Book Co., New York.

Watson, E. L. and Harper, J. C. 1988. Elements of Food Engineering. 2nd ed. Van Nostrand Reinhold Co., New York.